Multi-Agent Simulative Belief Ascription

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- *A* : "I do not like those who make the room messy".
- *B* : '*A* does not like people who make the room messy, and I am one of them'.
- *B* : "So *A* does not like me".
- *B* : Says to *C*, "*A* does not like me".

In the above scenario, we see what I will call *simulative belief ascription*. [13, 14] By definition, the ascribee does not genuinely hold such a belief; the ascriber merely treats it *as if* the ascribee did.

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3. **Multi-Agent AGM framework**.

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Problem(s):

1. While pragmatics helps us understand *why* we do this *conventionally*, it does not offer a *computationally robust* framework.

In the standard **Kripke-Hintikka** style (multi-agent) epistemic logic, an agent's beliefs are represented by an accessibility relation R on a set of possible worlds, $W = \{w_1, w_2, \ldots, w_n\}$. "Agent i believes p" is true at world w if p holds in all R_i-accessible worlds from w.

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2. **Fixed Access Relation**: The agent's doxastic possibilities are typically held fixed in a single model.
3. **Introspection and Revision**: Revising an agent's beliefs requires building a new (or globally modified) accessibility relation, or a new model altogether.

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1. **Simulative Operation**: Again, AGM is geared towards *genuine* beliefs, not *simulative* ones.
2. **Iterated Belief**: AGM primarily handles one-shot revision. It does not prescribe how beliefs evolve across multiple or nested updates.

Gerbrandy and Groeneveld [12, 1997], (also, Gerbrandy [11, 1999]) offered an *n*-agent framework which addresses iteration via a modular approach. In their setting, a world w is a triple $\langle u, b_i, b_j \rangle$.

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Solutions:

1. Aczel's *Anti-Foundation Axiom* [1, 1988](non-wellfounded set theory).
2. *Bisimilarity* to the Kripke-Hintikka model.

Cantwell [7, 2005] (and [8, 2007]) adopted Gerbrandy and Groeneveld's idea but developed a framework that does not rely on *non-wellfounded sets*. Crucially, the framework preserves a *modular representation* of possible worlds as $(n+1)$-tuples, $\langle u, b_1, b_2, \ldots, b_n \rangle$, where u determines belief-independent facts, and $b_1, \ldots, b_n$ represent each agent's belief state.

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This neatly represents *local changes* in the belief state of a single agent, e.g. from $\langle u, b_1, b_2, b_3 \rangle$ to $\langle u, b_1', b_2, b_3 \rangle$, without altering u (the belief-external facts) or other agents' states.

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- $\mathcal{C}$ is a function returning, for any agent i and $b \in \mathcal{B}_i$, a set of possible worlds.

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For a world $w = \langle u, b_1, \ldots, b_n \rangle$,

$$\mathrm{wst}(w) = u \quad \text{(gives the \textit{world-state} of } w\text{)},$$
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In his 2005 paper, Cantwell showed $\mathcal{F}$ can be represented by a standard Kripke system with n accessibility relations.

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The modular internal-world semantics for common learning is then combined with an AGM-style revision approach.

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$$\mathcal{B}^{sim}_{\langle i,j \rangle}, \quad b_{\langle i,j \rangle} \in \mathcal{B}^{sim}_{\langle i,j \rangle},$$

which denotes i's simulative belief states about j. An initial step in constructing such simulative states occurs after *common learning*, conceptually

$$w \xrightarrow{\oplus_N(\phi)} w' \xrightarrow{\mathsf{UpdSim}(\phi)} w''.$$

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$$\mathcal{B}^{sh}_{\langle j,i\rangle}, \quad b^{sh}_{\langle j,i\rangle} \in \mathcal{B}^{sh}_{\langle j,i\rangle},$$

denoting *shared states* between j and i, i.e. i's belief about j's belief. Informally, "j believes that i believes such-and-such".[2]

[2] This can arise via a *common sharing dynamic*, assumed always *sincere*, cf. Cantwell.

By introducing $\mathcal{B}^{sh}_{\langle j,i\rangle}$ and $\mathcal{B}^{sim}_{\langle i,j\rangle}$, the framework **localises** both shared and simulative beliefs by encapsulating them in separate compartments, preserving each agent's actual belief state $\mathcal{B}_i$.

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Thus, MASBA is defined:

Definition (1)

MASBA is a tuple

$$\langle W, U, \{\mathcal{B}_i\}_{1\leq i\leq n}, \mathcal{B}^{sh}_{\langle j,i\rangle}, \mathcal{B}^{sim}_{\langle i,j\rangle}, \mathcal{C}\rangle.$$

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MASBA generates accessibility relations R_i $(1 \leq i \leq n)$, where R_i is a binary relation on W such that

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Definition (3)

In MASBA, the accessibility relation for simulative beliefs $R_{\langle i,j \rangle}$ is a binary relation on W:

$$vR^{sim}_{\langle i,j \rangle} w \iff w \in \mathcal{C}\left(\mathrm{bst}^{sim}_{\langle i,j \rangle}(v)\right).$$

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A model $\mathfrak{M}$ consists of a MASBA structure plus a valuation function V, where for each propositional variable p, $V(p) \subseteq U$. Truth is evaluated at possible worlds:

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6. $w \vDash B^{sim}_{\langle i,j\rangle}\phi$ iff for each $w' \in \mathcal{C}\left(\mathrm{bst}^{sim}_{\langle i,j\rangle}(w)\right)$, $w' \vDash \phi$.
7. $w \vDash B^{sh}_{\langle i,j\rangle}\phi$ iff for each $w' \in \mathcal{C}\left(\mathrm{bst}^{sh}_{\langle i,j\rangle}(w)\right)$, $w' \vDash \phi$.

The deductive system of MASBA consists of a $KD45$ system for the operator B_i, and a K system for $B^{sh}_{\langle j,i\rangle}$ and $B^{sim}_{\langle i,j\rangle}$:

1. Tautologies,
2. (K) $B_i(\phi \to \psi) \to (B_i\phi \to B_i\psi)$, similarly for $B^{sh}_{\langle i,j\rangle}$ and $B^{sim}_{\langle i,j\rangle}$,
3. (D) $\neg(B_i\phi \wedge B_i\neg\phi)$,
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The framework is *sound* and *complete*[3] showing that MASBA is fully representable in a standard Kripke-Hintikka system.

[3] A proof will appear on my website soon.

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A simple expansion occurs as

$$\mathcal{C}\big(b^{sim}_{\langle i,j\rangle} + \mathcal{C}(b^{sh}_{\langle j,i\rangle})\big) = \Big\{+^{sim}_{\langle i,j\rangle}\big(b^{sh}_{\langle j,i\rangle}, w\big) \mid w \in \mathcal{C}\big(b^{sim}_{\langle i,j\rangle}\big)\Big\}.$$

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If $\mathcal{C}\left(b^{sh}_{\langle j,i\rangle}\right) \cap \mathcal{C}\left(b^{sim}_{\langle i,j\rangle}\right) \cap \phi \neq \emptyset$, $\gamma_{(b^{sh},\, b^{sim})}(\phi) = \mathcal{C}\left(b^{sh}_{\langle j,i\rangle}\right) \cap \mathcal{C}\left(b^{sim}_{\langle i,j\rangle}\right) \cap \phi$.

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When multiple compartments take part simultaneously, we can modify this selection function accordingly.

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That is, $*_{\langle i,j\rangle}$ is a *simulative belief revision function*, adding $\mathcal{C}(b_j)$ with a minimal revision of $\mathrm{bst}^{sim}_{\langle i,j\rangle}(w)$:

$$\mathcal{C}\left(b^{sim}_{\langle i,j\rangle} * \mathcal{C}(b_j)\right) = \left\{ *_{\langle i,j\rangle}\left(\mathcal{C}(b_j),\ w\right) \mid w \in \gamma_{(b^{sim}_{\langle i,j\rangle})}\left(\mathcal{C}(b_j)\right) \right\}.$$

Here, the agent j revises the simulative belief state $b^{sim}_{\langle i,j\rangle}$ with respect to j's *own* belief state b_j.

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> **Tennis**: *Ann is a six-year-old girl whom Pete, an expert in tennis pedagogy, has never met and whose existence he is unaware of. Pete believes that any six-year-old can learn tennis in ten lessons. Jane, Ann's aunt, knows Pete's views and wants to encourage Ann's father, Jim, to enrol Ann in tennis lessons. During conversation with Jim, Jane asserts:*

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1. Pete believes that 'every 6-year-old can learn to play tennis in ten lessons'. This is a **genuine belief** in Pete's belief state, b_i.
2. Jane knows Pete's belief and applies it to Ann, even though Pete is **unaware of Ann's existence**.
3. Jane ascribes the belief 'Ann can learn tennis in ten lessons' to Pete, when talking to Ann's father, Jim.

Formal Representations in Masba

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b_i Pete's genuine belief state.

b_j Jane's genuine belief state.

$b^{sh}_{\langle i,j \rangle}$ Pete's **shared belief state** to Jane.

$b^{sim}_{\langle j,i \rangle}$ Pete's **simulative belief state** about Ann that Jane has.

1. Pete's Belief State (Agent i)

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- Pete's belief state b_i includes the general belief:

$$b_i \vDash \forall x \begin{pmatrix} \text{x is six years old, and x can learn how to play} \\ \text{tennis in ten lessons.} \end{pmatrix}$$

1. **Pete's Belief State** (Agent i)

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- (In)Formally:

$$b_i \vDash \{\phi \mid \phi \text{ is consistent with Pete's belief}\}$$

2. Jane's Belief (Agent j)

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   - Jane's belief state b_j includes two key pieces of information:

$$b_j \vDash \exists y S(y) \quad \text{(Ann exists, and Ann is six years old)}$$
$$b_j \vDash b_i \vDash (\forall x \; x \text{ can learn to play tennis})$$

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- (In)Formally, Jane's belief state is:

$$b_j = \{\psi, \chi \mid \psi \text{ is consistent with Jane's belief, and } \chi = \text{(Ann is six years old)}\}$$

3. Shared Belief ($b^{sh}_{\langle i,j \rangle}$)

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- This would be something like:

$$b^{sim}_{\langle j,i \rangle} \vDash (\text{If Pete knew Ann is six years old, } \ldots)$$

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- After $\oplus_N(\cdot)$, Jane updates $b^{sim}_{\langle j,i \rangle}$.

3 Shared State Update:

- Pete tells Ann about his belief, prompting Jane to construct a shared belief about Pete:

$$b^{sh}_{\langle i,j \rangle} \vDash \phi$$

5. Masba Dynamics in Action (Continued)

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4 Simulative State Update:

- Jane updates her simulative state about Pete by first including the **shared state**:

$$b^{sim}_{\langle j,i\rangle} \leftarrow b^{sh}_{\langle i,j\rangle}$$

- Followed by the revision step:

$$b^{sim}_{\langle j,i\rangle} \leftarrow b^{sh}_{\langle i,j\rangle} * \mathcal{C}(b_j)$$

- This ensures Jane's simulative states of Pete are consistent with her own belief state.

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- **Integrity of Each Belief Compartment:** The world is represented as a tuple:

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- Pete's belief is in b_i, and the simulative state is in a separate compartment, $b^{sim}_{\langle j,i \rangle}$.

MASBA, an extension of $\mathcal{F}$ incorporating *simulative* and *shared* belief states, provides a modular internal-worlds semantics for simulative belief ascriptions between agents. By treating a world as

$$w = \langle u,\ b_1, \ldots, b_n,\ b^{sh}_{\langle i,j\rangle (1\leq i,j\leq n \,|\, i\neq j)},\ b^{sim}_{\langle i,j\rangle (i\leq i,j\leq n \,|\, i\neq j)}\rangle,$$

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4. Incorporating AGM-style revision for simulative belief ascriptions as well.

Thank you!

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